III. "Memoir on the Symmetric Functions of the Roots of certain Systems of two Equations." By ARTHUR CAYLEY, Esq., F.R.S. Received December 18, 1856.

(Abstract.)

The author defines the term *roots* as applied to a system of $n-1$ equations $\phi=0$, $\psi=0$, &c., where ϕ, ψ, &c., are quantics (*i. e.* rational and integral homogeneous functions) of the n variables $(x, y, z, ..)$ and the terms *symmetric functions* and *fundamental symmetric functions* of the roots of such a system; and he explains the process given in Professor Schläfle's memoir, "Ueber die Resultante eines Systemes mehrerer algebraischer Gleichungen," Vienna Transactions, t. iv. (1852), whereby the determination of the symmetric functions of any system of $(n-1)$ equations, and of the resultant of any system of n equations is made to depend upon the very simple question of the determination of the resultant of a system of n equations, all of them, except one, being linear. The object of the memoir is then stated to be the application of the process to two particular cases, viz. to obtaining the expressions for the simplest symmetric functions, after the fundamental ones of the following systems of two ternary equations, viz. first, a linear equation and a quadratic equation; and secondly, a linear equation and a cubic equation; and the author accordingly obtains expressions, as regards the first system, for the fundamental symmetric functions or symmetric functions of the first degree in respect to each set of roots, and for the symmetric functions of the second and third degrees respectively, and as regards the second system, for the fundamental symmetric functions or symmetric functions of the first degree, and for the symmetric functions of the second degree in respect to each set of roots.

IV. "Memoir on the Resultant of a System of two Equations." By ARTHUR CAYLEY, Esq., F.R.S. Received December 18, 1856.

(Abstract.)

The resultant of two equations such as

$$(a, b, ..\between x, y)^m=0$$
$$(p, q, ..\between x, y)^n=0$$

is, it is well known, a function homogeneous in regard to the coefficients of each equation separately, viz. of the degree n in regard to the coefficients $(a, b, ..)$ of the first equation, and of the degree m in regard to the coefficients $(p, q, ..)$ of the second equation; and it is natural to develope the resultant in the form $k\text{AP}+k'\text{A}'\text{P}'+$ &c., where A, A′, &c. are the combinations (powers and products) of the degree n in the coefficients $(a, b, ..)$, P, P′, &c. are the combinations of the degree m in the coefficients $(p, q, ..)$, and k, k', &c. are mere numerical coefficients. The object of the present memoir is to show how this may be conveniently effected, either by the method of symmetric functions, or from the known expression of the resultant in the form of a determinant, and to exhibit the developed expressions for the resultant of two equations, the degrees of which do not exceed 4. With respect to the first method, the formula in its best form, or nearly so, is given in the 'Algebra' of Meyer Hirsch, and the application of it is very easy when the necessary tables are calculated: as to this, see my "Memoir on the Symmetric Functions of the Roots of an Equation." But when the expression for the resultant of two equations is to be calculated without the assistance of such tables, it is, I think, by far the most simple process to develope the determinant according to the second of the two methods.

V. "Memoir on the Symmetric Functions of the Roots of an Equation." By ARTHUR CAYLEY, Esq., F.R.S. Received December 18, 1856.

(Abstract.)

There are contained in a work, which is not, I think, so generally known as it deserves to be, the 'Algebra' of Meyer Hirsch, some very useful tables of the symmetric functions up to the tenth degree of the roots of an equation of any order. It seems desirable to join to these a set of tables, giving reciprocally the expressions of the powers and products of the coefficients in terms of the symmetric functions of the roots. The present memoir contains the two sets of tables, viz. the new tables distinguished by the letter (a), and the tables of Meyer Hirsch distinguished by the letter (b); the memoir contains